Topic 04 – Factorising Quadratics (Solutions)

Section 1: Routine Factorisation Practice

Easy

1. $(a-4)(a-2)$	6. $(a-3)(a-4)$	11. $(a-1)^2$	16. $(a-4)(a-5)$
2. $(a+2)(a+5)$	7. $(a+4)(a-2)$	12. $(a-4)(a-1)$	17. $(a-4)(a+3)$
3. $(a-2)(a-1)$	8. $(a+3)(a+5)$	13. $(a-3)(a+1)$	18. $(a+4)(a+5)$
4. $(a-4)(a-1)$	9. $(a+3)(a+2)$	14. $(a+4)(a+2)$	19. $(a-5)(a+2)$
5. $(a+2)(a-1)$	10. $(a+3)^2$	15. $(a-4)(a+5)$	20. $(a-4)(a+2)$

Harder

1. $(3a+4)(a+1)$	6. $(4a-3)(a-3)$	11. $(4a+3)(a-2)$	16. $(3a-5)(a+1)$
2. $(4a-3)(a-3)$	7. $(4a+1)(a-4)$	12. $(2a-3)(a-2)$	17. $(4a-1)(a-4)$
3. $(3a-2)(a+1)$	8. $(5a-1)(a+3)$	13. $(3a-5)(a+3)$	18. $(3a-2)(a+5)$
4. $(4a+3)(a-5)$	9. $(3a-2)(a+2)$	14. $(3a+2)(a+1)$	19. $(2a-3)(a-3)$
5. $(2a-5)(a-4)$	10. $(5a+4)(a-2)$	15. $(3a-4)(a+2)$	20. $(3a+2)(a-3)$

Section 2: Problem Solving

Q1, (Jan 2007, Q9ii)

Factorise $x^2 - 4$ and $x^2 - 5x + 6$.

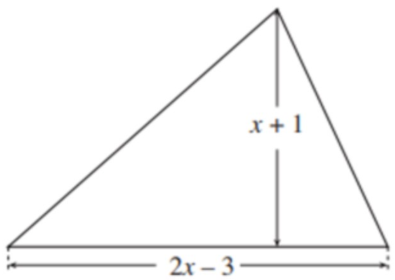
Hence express $\dfrac{x^2-4}{x^2-5x+6}$ as a fraction in its simplest form. [3]

$x^2 - 4 = (x+2)(x-2)$ $\qquad$ $x^2 - 5x + 6 = (x-2)(x-3)$

$$\therefore \frac{x^2-4}{x^2-5x+6} = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} = \boxed{\frac{x+2}{x-3}}$$

Q2, (Jun 2007, Q10)

The triangle shown in Fig. 10 has height $(x+1)$ cm and base $(2x-3)$ cm. Its area is 9 cm^2.



Not to scale

Fig. 10

(i) Show that $2x^2 - x - 21 = 0$. [2]

(ii) By factorising, solve the equation $2x^2 - x - 21 = 0$. Hence find the height and base of the triangle. [3]

i) Area: $\frac{1}{2}(2x-3)(x+1) = 9 \implies (2x-3)(x+1) = 18$

$\implies 2x^2 + 2x - 3x - 3 = 18 \implies 2x^2 - x - 3 = 18$

$\implies 2x^2 - x - 21 = 0$

ii) $(2x-7)(x+3) = 0 \implies 2x - 7 = 0$ or $x + 3 = 0$

$\implies 2x = 7 \implies x = \frac{7}{2}$ or $\cancel{x = -3}$

$\boxed{\therefore \text{ base} = 2\left(\tfrac{7}{2}\right) - 3 = 4 \quad \text{height} = \tfrac{9}{2}}$

Invalid as leads to negative height

Q3, (Jan 2008, Q2)

Factorise and hence simplify $\dfrac{3x^2 - 7x + 4}{x^2 - 1}$. [3]

$$\frac{(3x-4)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \boxed{\frac{3x-4}{x+1}}$$

Q4, (Jun 2008, Q3i)

Solve the equation $2x^2 + 3x = 0$. [2]

$x(2x+3) = 0 \Rightarrow x = 0$ or $2x + 3 = 0$

$\Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

$$\boxed{\therefore\ x = 0 \text{ or } x = -\frac{3}{2}}$$

Q5, (Jun 2008, Q9)

Solve the equation $y^2 - 7y + 12 = 0$.

Hence solve the equation $x^4 - 7x^2 + 12 = 0$. [4]

$(y-3)(y-4) = 0 \Rightarrow y = 3$ or $y = 4$

Notice in $x^4 - 7x^2 + 12 = 0$. y has been replaced with x^2

$\therefore\ x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ or $x^2 = 4 \Rightarrow x = \pm 2$

$$\Rightarrow \boxed{x = \pm\sqrt{3} \text{ or } \pm 2}$$

Q6, (Jun 2010, Q10i, ii)

(i) Solve, by factorising, the equation $2x^2 - x - 3 = 0$. **[3]**

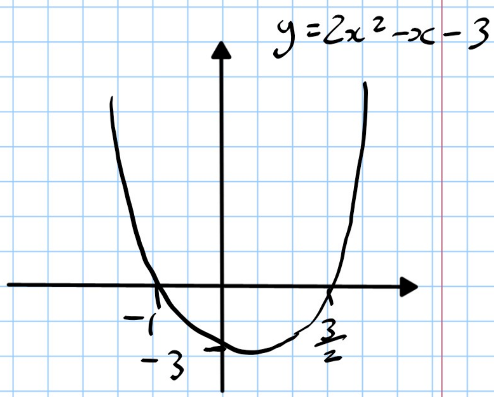
(ii) Sketch the graph of $y = 2x^2 - x - 3$. **[3]**

i/ $(2x-3)(x+1) = 0$

$\Rightarrow 2x - 3 = 0$ or $x + 1 = 0$

$\Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$ $\Rightarrow x = -1$

$$\boxed{\therefore\ x = -1 \text{ or } \frac{3}{2}}$$



Q7, (Jan 2011, Q9)

Fig. 9 shows a trapezium ABCD, with the lengths in centimetres of three of its sides.

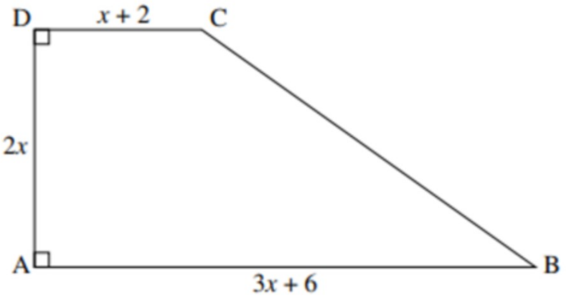


Fig. 9

This trapezium has area $140\,\text{cm}^2$.

(i) Show that $x^2 + 2x - 35 = 0$. **[2]**

(ii) Hence find the length of side AB of the trapezium. **[3]**

i/ $\frac{1}{2}(2x)(x+2 + 3x+6) = 140$

$\Rightarrow x(4x+8) = 140 \Rightarrow 4x^2 + 8x - 140 = 0$

$\Rightarrow x^2 + 2x - 35 = 0$

ii/ $(x+7)(x-5) = 0 \Rightarrow \cancel{x = -7}$ or $x = 5$

(Cannot have neg. length)

$\Rightarrow$ $\boxed{\text{length of AB} = 3(5) + 6 = 21}$

Q8, (Jun 2012, Q4)

Factorise and hence simplify the following expression.

$$\frac{x^2 - 9}{x^2 + 5x + 6}$$ **[3]**

$$\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x+2)} = \frac{x-3}{x+2}$$